with a natural crack, which lay for about a year under room conditions. Under the same conditions as in the experiment, the crack opened up with deformations much smaller than $0.39 \cdot 10^{-3}$. This fact is evidently explained by the relaxation of induced stresses.

We note in conclusion that for small deformations, a natural macrocrack does not open up completely, opening up of the crack is opposed by the compressive induced stresses, and since a loosening zone evidently accompanies the formation of a natural crack in any material, the results of the present work are apparently also valid for a wider class of materials.

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EXISTENCE OF SOLUTIONS IN DYNAMICS PROBLEMS OF ONE-DIMENSIONAL PLASTIC STRUCTURES

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UDC 539.214+539.374+517.9

The distinctive feature in the formulations of elastic–plastic and rigid–plastic problems is the presence of an inequality connecting the plastic strain rate and the magnitude of the running stresses. This inequality, called the Mises maximum principle, includes r plastic strain rate components (r depends on the dimensionality of the problem), where it is arranged so that it actually replaces r equations and the system of governing relationships is hence closed. Therefore, it turns out that upon the assignment of initial and boundary conditions, the rates and stresses are determined at each point, and moreover uniquely. Let us note that a corollary of the mentioned inequality that describes the proportionality between the plastic strain component and the components of the flow surface gradient is often used in finding the approximate solutions (by a numerical or analytic method). As a rule, this results in openness of the system of equations. In this sense the utilization of the maximum principle in its initial form is more preferable despite the fact that the inequality itself is a corollary of the more general Drucker postulate. In particular, formulation of the problem by using the inequality was examined in [1], which permitted setting up the solvability of the three-dimensional dynamic elastic–plastic problem.

Generalized stresses (forces, moments, etc.) and strain rates of the middle surface take part in the formulation of elastic–plastic and rigid–plastic problems for thin-walled structures of the shell, plate, and beam type. They are also interrelated by using inequalities [2, 3]. Definite progress has been achieved in the investigation of problems of this kind from the viewpoint of an approximate description of the strain processes. This concerns the case of one space variable especially (see the survey [4]). However, despite the large number of papers on this topic, in practice there are no results referring to the investigation of the correctness in the formulations of such problems. Boundary-value problems for one-dimensional elastic–plastic and rigid–plastic structures are considered in this paper, and results on solvability are formulated.

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 150-156, March-April, 1983. Original article submitted April 28, 1982.

0021-8944/83/2402-0274$07.50

Taking account of shear and rotational inertia, the beam dynamics equations have the form [5]

$$l_1 \equiv u_t - n_x - f_1 = 0,\ l_2 \equiv v_t - m_x + q - f_2 = 0,\quad l_3 \equiv w_t - q_x - f_3 = 0, \tag{1}$$

where m, n, q are the bending moment, force, and transverse force, respectively; u, velocity of points of the beam middle line along the Ox axis; w, deflection rate; v, rate of change of the angle of rotation of the normal to the Ox axis; and f_i, i = 1, 2, 3, external loads.

Let us first examine an ideal rigid–plastic problem. Let the flow condition have the form $\Phi(n, m, q) = 1$. It is assumed that Φ is a convex continuous function. The plastic strain rate is determined from the flow law associated with the function Φ. The corresponding maximum principle has the form [2]

$$u_x(\bar{n} - n) + v_x(\bar{m} - m) + (w_x + v)(\bar{q} - q) \leqslant 0 \tag{2}$$

for any $(\bar{n}, \bar{m}, \bar{q})$ such that $\Phi(\bar{n}, \bar{m}, \bar{q}) \leqslant 1$.

It turns out that the system of equations (1) and the inequality (2) are a system of closed relationships in the sense that for given initial and boundary conditions the existence of the functions u, v, w, n, m, q can be established. To formulate the result exactly, a set of notations must be introduced. We firstly determine the closed convex set related to the flow condition

$$K = \{(n, m, q) | n, m, q \in L^2(a, b),\ \Phi(n, m, q) \leqslant 1 \text{ almost everywhere in } (a, b)\}.$$

We consider that $0 \in K$. We denote by $H_a^s(a, b)$ the space of Sobolev functions that have square summable generalized derivatives in the interval (a, b) to the order of s inclusive and equal 0 for $x = a$ to order $s - 1$. We define $H_b^s(a, b)$ analogously. Also let $Q = (a, b) \times (0, T)$, $T > 0$. For brevity, we shall use the notation P = (u, v, w) and R = (n, m, q).

Formulation of the problem in the form of integral identities and inequalities in which the derivatives with respect to the space variable are dumped in the trial function is most natural. This latter circumstance is related to the fact that all first derivative with respect to x exist as functions from the space $L^2(Q)$.

The following result that refers to the dynamic rigid–plastic problem for a beam on the basis of a model taking into account the shear and inertia of rotation is valid.

THEOREM 1. Let $f_i, f_{it} \in L^2(Q)$, i = 1, 2, 3. Then there exist and, moreover, are unique functions P = (u, v, w), R = (n, m, q) that satisfy the following relationships

$$\int_a^b (u_t h + n h_x - f_1 h)\, dx = 0 \quad \forall h \in H_a^1(a, b); \tag{3}$$

$$\int_a^b (v_t h + m h_x + q h - f_2 h)\, dx = 0 \quad \forall h \in H_a^1(a, b); \tag{4}$$

$$\int_a^b (w_t h + q h_x - f_3 h)\, dx = 0 \quad \forall h \in H_a^1(a, b); \tag{5}$$

$$\int_a^b \{u(\bar{n}_x - n_x) + v(\bar{m}_x - m_x) + w(\bar{q}_x - q_x) - v(\bar{q} - q)\}\, dx \geqslant 0 \quad \forall \bar{R} = (\bar{n}, \bar{m}, \bar{q}) \in K \cap H_b^1(a, b), \tag{6}$$

where P = 0 for t = 0, $R(t) \in K$ almost everywhere on (0, T), $P, P_t \in L^\infty(0, T; L^2(a, b))$, $R \in L^\infty(0, T; H_b^1(a, b))$.

The inclusion $R(t) \in K$ means that the generalized stresses do not exceed the flow limit.

A scheme for the proof of this theorem is presented below. Let B denote the penalty operator related to the set K and acting from $[L^2(a, b)]^3$ to $[L^2(a, b)]^3$. It can be considered that the operator B is continuous and monotonic. Let $\varepsilon, \delta > 0$ be fixed numbers. We consider the auxiliary problem with the penalty

$$l_i = 0,\ i = 1, 2, 3, \tag{7}$$

$$l_4 \equiv \varepsilon n_t - u_x + \delta^{-1}B(R)_1 = 0; \tag{8}$$

$$l_5 \equiv \varepsilon m_t - v_x + \delta^{-1}B(R)_2 = 0; \tag{9}$$

$$l_6 \equiv \varepsilon q_t - w_x - v + \delta^{-1}B(R)_3 = 0; \tag{10}$$

$$P = 0,\ R = 0 \text{ for } \quad t = 0; \tag{11}$$

$$P = 0 \text{ for } x = a,\ R = 0 \text{ for } x = b. \tag{12}$$

Here $B(R)_i$ denotes the components of the penalty operator. Insertion of the parameters ε is actually equivalent to the fact that the initial rigid-plastic problem is approximated by an elastic-plastic problem (with the Young's modulus ε^{-1}). In turn, the latter is approximated by a problem with the penalty (7)-(12). Let us first establish its sensitivity. This can be done by using the Galerkin method by selecting the bases $\{\varphi_j\}$, $\{\psi_j\}$, j = 1, 2, 3, ..., in the spaces $H^1_a(a, b)$ and $H^1_b(a, b)$, respectively. The approximate solution should be sought in the form

$$P^s(t) = \sum_{i=1}^{s} a_i^s(t)\varphi_i, \quad R^s(t) = \sum_{i=1}^{s} b_i^s(t)\psi_i,$$

where the three-component vector-functions $a_i^s(t)$, $b_i^s(t)$ are determined from the following systems of ordinary differential equations

$$\int_a^b l_i^s \varphi_j dx = 0,\ i = 1, 2, 3; \quad \int_a^b l_i^s \psi_j dx = 0,\ i = 4, 5, 6;\ j = 1, 2, \ldots, s.$$

We consider the derivatives with respect to the space variable to be dumped in the basis functions when writing these equations (exactly as in writing the identities (3)-(5)). The initial data for P^s, R^s are zero. An a priori estimate of problem (7)-(12) has the following form

$$\max_{0 \leqslant t \leqslant T} \{\|P(t)\| + \|P_t(t)\| + \varepsilon^{1/2}\|R(t)\| + \varepsilon^{1/2}\|R_t(t)\|\} \leqslant c_1. \tag{13}$$

The constant c_1 depends only on the function f_i (i = 1, 2, 3) and T, and $\|\cdot\|$ is the norm in $L^2(a, b)$. An estimate is obtained by using multiplication of (7)-(10), respectively, by u, v, w, n, m, q, subsequent differentiation with respect to t, and multiplication by u_t, v_t, w_t, n_t, m_t, q_t and taking account of the monotonicity of the penalty operator. Reproduction of the a priori estimate for the Galerkin approximation is realized in the usual manner. Thus the estimate (13) might be considered valid for P^s, R^s with a constant c_1 independent of s. This circumstance permits making a conclusion about the solvability of the Galerkin equations in the interval (0, T) and realizing the passage to the limit there as $s \to \infty$. Let us emphasize that although the limit equations will be satisfied in the sense of integral identities (analogous to (3)-(5)) in which the derivatives with respect to x are dumped in the trial functions, it can be considered that $P \in L^\infty(0, T; H^1_a(a, b))$, $R \in L^\infty(0, T; H^1_b(a, b))$ (because of the validity of the equations in the sense of distributions). Actually, P and R depend on the parameters ε, δ. In this connection, we note that an estimate of the function P in the space $L^\infty(0, T; H^1_a(a, b))$ is not uniform in ε, δ. The fact that $B(R^s)$ converges weakly to B(R) in $L^2(Q)$ is confirmed by using the monotonicity of the operator B.

The next stage in the discussion is the passage to the limit as $\varepsilon \to 0$ in (7)-(12). We equip the solution with the symbols ε, δ. The following estimates hold for this solution

$$\max_{0 \leqslant t \leqslant T} \{\|P^{\varepsilon\delta}(t)\| + \|P_t^{\varepsilon\delta}(t)\|\} \leqslant c_2, \quad \max_{0 \leqslant t \leqslant T} \|R^{\varepsilon\delta}(t)\|_{H_b^1(a,b)} \leqslant c_3 \tag{14}$$

with the constants in the right sides independent of ε, δ. As before, the first is obtained by using multiplication of (7)-(10) and the equations obtained by differentiation with respect to t, while the second follows from (7) directly with the first taken into account. Hence, it can be considered that as $\varepsilon \to 0$ $P^{\varepsilon\delta}$, $P_t^{\varepsilon\delta} \to P^\delta$, $P_t^{\delta *}$ weakly in $L^\infty(0, T; L^2(a, b))$, $R^{\varepsilon\delta} \to R^{\delta *}$ weakly in $L^\infty(0, T; H^1_b(a, b))$. After the passage to the limit as $\varepsilon \to 0$ we obtain ($\varepsilon R_t^{\varepsilon\delta} \to 0$ weakly in $L^2(Q)$)

$$u_t^\delta - n_x^\delta = f_1, \quad v_t^\delta - m_x^\delta + q^\delta = f_2, \quad w_t^\delta - q_x^\delta = f_3; \tag{15}$$

$$-u_x^\delta + \delta^{-1}B(R^\delta)_1 = 0, \quad -v_x^\delta + \delta^{-1}B(R^\delta)_2 = 0, \quad -w_x^\delta - v^\delta + \delta^{-1}B(R^\delta)_3 = 0. \tag{16}$$

Taking account of the available smoothness, it can be asserted that the boundary condition

$R^\delta = 0$ at $x = b$ is satisfied in the sense of L^2 while the condition $P^\delta = 0$ at $x = a$ is satisfied in the weak sense and is contained in the appropriate integral identities. Moreover, the initial condition $P^\delta = 0$ at $t = 0$ is satisfied in the sense of L^2.

The concluding step in the discussion is the passage to the limit in (15) and (16) as $\delta \to 0$. Since estimates (14) are uniform in δ, it can then be considered

$$\max_{0 \leqslant t < T} \left\{ \| P^\delta (t) \| + \| P_t^\delta (t) \| + \| R^\delta (t) \|_{H_b^1(a,b)} \right\} \leqslant c_4.$$

The constant c_4 is independent of δ. Let the subsequence P^δ, R^δ denoted as before possess the property that as $\delta \to 0$ the convergence of P^δ, P_t^δ to the functions P, P_t* holds weakly in the space $L^\infty(0, T;\ L^2(a, b))$ and R^δ to R* weakly in $L^\infty(0, T;\ H_b^1(a, b))$. The passage to the limit in (15) is realized in the usual way, and we proceed thus in (16). We take any function $\bar{R} \in L^2(0, T; K \cap H_b^1(a, b))$. Then it follows from (16) (we emphasize here that the equations are satisfied in the sense of integral identities in which substitution of the trial functions, $\bar{n} - n^\delta$, $\bar{m} - m^\delta$, $\bar{q} - q^\delta$, respectively, is allowable)

$$\int_a^b \{ u^\delta (\bar{n}_x - n_x^\delta) + v^\delta (\bar{m}_x - m_x^\delta) + w^\delta (\bar{q}_x - q_x^\delta) - v^\delta (\bar{q} - q^\delta) \} \, dx \geqslant 0.$$

Let us substitute n_x^δ, m_x^δ, q_x^δ from (15) here, and let us integrate with respect to t between 0 and T. We obtain

$$\int_0^T \int_a^b (u^\delta \bar{n}_x + v^\delta \bar{m}_x + w^\delta \bar{q}_x - v^\delta \bar{q}) \, dx dt + \int_0^T \int_a^b (f_1 u^\delta + f_2 v^\delta + f_3 w^\delta) \, dx dt \geqslant \frac{1}{2} \left(\| u^\delta (T) \|^2 + \| v^\delta (T) \|^2 + \| w^\delta (T) \|^2 \right).$$

Because of the mentioned convergence and the inequality $\lim ||P^\delta(T)|| \geqslant ||P(T)||$, we pass here to the lower limit. Hence

$$\int_0^T \int_a^b \{ u (\bar{n}_x - n_x) + v (\bar{m}_x - m_x) + w (\bar{q}_x - q_x) - v (\bar{q} - q) \} \, dx dt \geqslant 0$$

$$\forall \bar{R} \in L^2 (0, T;\ K \cap H_b^1 (a, b)).$$

Hence, inequality (6) is easily obtained for almost all $t \in (0, T)$. The imbedding $R(t) \in K$ can also be set up almost everywhere from (16), where we do not present the foundation. We just note that the monotonicity of the operator B is used here.

Now we prove the uniqueness of the solution in the class of dunctions under consideration. The equations

$$u_t - n_x = 0,\ v_t - m_x + q = 0,\ w_t - q_x = 0 \tag{17}$$

are satisfied for the difference of two possible solutions $P = P_1 - P_2$ and $R = R_1 - R_2$. We substitute the trial function $\bar{R} = \bar{R}(t) = R_2(t)$ for the solution P_1, R_1 in inequality (6), and $\bar{R} = \bar{R}(t) = R_1(t)$ for the solution P_2, R_2. Combining the relationships obtained, and taking (17) into account, we obtain

$$\frac{d}{dt} \int_a^b (u^2 + v^2 + w^2) \, dx \leqslant 0.$$

Hence $u = v = w = 0$ follows. Then we conclude from (17) that also $n = m = q = 0$.

Therefore, the existence of the solution and its uniqueness have been established in the ideal elastic–plastic problem for the beam model under consideration. The existence of the solution will be established if the passage to the limit as $\delta \to 0$ is realized in the problem (7)-(12) for $\varepsilon = 1$. The foundation for the possibility of such a passage is executed approximately the same as in Theorem 1 for the case of the rigid–plastic problem. Let us introduce a notation for the bilinear form

$$C (Q, R) = \int_a^b (zn + gm + hq) \, dx, \quad Q = (z, g, h), \quad R = (n, m, q)$$

and let us formulate the corresponding result in the form of a theorem.

THEOREM 2. Let f_i, $f_{it} \in L^2(Q)$, $i = 1, 2, 3$

$$P_0 = (u_0, v_0, w_0) \in H_a^1(a, b), \quad R_0 = (n_0, m_0, q_0) \in H_b^1(a, b) \cap K.$$

Then there exist functions, and unique ones at that, $P = (u, v, w)$ and $R = (n, m, q)$ that satisfy the relationships (3)-(5) and the inequality

$$C(R_t, \bar{R} - R) + \int_a^b [u(\bar{n}_x - n_x) + v(\bar{m}_x - m_x) + w(\bar{q}_x - q_x)$$
$$- v(\bar{q} - q)]\, dx \geqslant 0 \qquad \forall \bar{R} = (\bar{n}, \bar{m}, \bar{q}) \in K \cap H_b^1(a, b),$$

where

$$P = P_0, \ R = R_0 \text{ при } t = 0; \ R(t) \in K \text{ almost everywhere in } (0, T);$$
$$P, P_t, R_t \in L^\infty(0, T; L^2(a, b)), \ R \in L^\infty(0, T; H_b^1(a, b)).$$

Now, let us examine the model of a beam without shear and rotational inertia. The equilibrium equations here have the form $u_t - n_x = f_1$, $w_t - m_{xx} = f_2$. The functions u, w, n, m have the same meaning as before, and f_1, f_2 are the longitudinal and transverse loads. We assume that the flow condition has the form $\Phi_1(n, m) = 1$, and Φ_1 is a convex and continuous function of two variables such that $\Phi_1(0, 0) \leqslant 1$. We consider the plastic flow rate determined by the vector $(u_x - n_t, -w_{xx} - m_t)$ associated with the function Φ_1. As before, we introduce the set

$$K_1 = \{(n, m) | n, m \in L^2(a, b), \ \Phi_1(n, m) \leqslant 1 \text{ almost everywhere in } (a, b)\}.$$

The following result, referring to the dynamics of an ideal elastic-plastic beam without taking account of shear and rotational inertia, will hold.

THEOREM 3. We assume that f_i, $f_{it} \in L^2(Q)$, $i = 1, 2, 3$

$$u_0 \in H_a^1(a, b), \ w_0 \in H_a^2(a, b), \ n_0 \in H_b^1(a, b), \ m_0 \in H_b^2(a, b), \ (n_0, m_0) \in K_1.$$

Then unique functions u, w, n, m exist that possess the properties

$$\int_a^b (u_t h + n h_x)\, dx = \int_a^b f_1 h\, dx \qquad \forall h \in H_a^1(a, b); \tag{18}$$

$$\int_a^b (w_t g - m g_{xx})\, dx = \int_a^b f_2 g\, dx \qquad \forall g \in H_a^2(a, b); \tag{19}$$

$$\int_a^b [n_t(\bar{n} - n) + m_t(\bar{m} - m) + u(\bar{n}_x - n_x) + w(\bar{m}_{xx} - m_{xx})]\, dx \geqslant 0 \tag{20}$$

$$\forall (\bar{n}, \bar{m}) \in K_1 \cap V_1,$$
$$u, u_t, w, w_t \in L^\infty(0, T; L^2(a, b)),$$
$$n \in L^\infty(0, T; H_b^1(a, b)), \ m, m_{xx} \in L^\infty(0, T; L^2(a, b)),$$
$$u = u_0, \ w = w_0, \ n = n_0, \ m = m_0 \text{ for } t = 0,$$

where $(n(t), m(t)) \in K$ almost everywhere in $(0, T)$. Here $V_1 = \{(n, m) | n \in H_b^1(a, b), m, m_{xx} \in L^2(a, b), m = m_x = 0$ for $x = b$ (in sense of $H^{-1/2}$ and $H^{-3/2}$, respectively)$\}$.

The initial problem approximates the problem with a penalty in the proof of this theorem. The boundary conditions for u, w, n, m are contained in the identities (18) and (19) and the inequality (20).

In conclusion, we consider the case of a cylindrical shell with axial symmetry whose equilibrium equation can be written in the form $w_t - m_{xx} - N = f$, where w is the deflection velocity, m is the axial bending moment, N is the circumferential force per unit length, and f is the transverse load. Let the flow condition be described by the function Φ_2 so that $\Phi_2(m, N) < 1$ corresponds to the elastic state and $\Phi_2(m, N) = 1$ to the plastic state. We consider $\Phi_2(0, 0) \leqslant 1$, and Φ_2 is continuous and convex in the set of variables. The plastic flow velocity vector has the components $(-w_{xx} - m_t, -w - N_t)$ and is associated with the mentioned flow condition. Let $K_2 = \{(m, N) | m, N \in L^2(a, b), \ \Phi_2(m, N) \leqslant 1$ almost everywhere in $(a, b)\}$, $V_2 = \{(m, N) | m, m_{xx} \in L^2(a, b), m = m_x = 0$ for $x = b$ (in the sense of $H^{-1/2}$ and $H^{-3/2}$, respectively), $N \in L^2(a, b)\}$.

The solution of the dynamic, ideal elastic–plastic problem for a cylindrical shell exists and is unique. Let us present a formulation of the corresponding result.

THEOREM 4. We assume that f, $f_t \in L^2(Q)$, $w_0 \in H_a^2(a, b)$, $m_0 \in H_b^2(a, b)$, $N_0 \in L^2(a, b)$, $(m_0, N_0) \in K_2$. Then there exist unique functions w, m, N that satisfy the relationships

$$w,\ w_t,\ m,\ m_t,\ N,\ N_t \in L^\infty(0,\ T;\ L^2(a,\ b)), \tag{21}$$

$$\int_a^b (w_t h - m h_{xx} - N h)\, dx = \int_a^b f h dx \qquad \forall h \in H_a^2(a, b);$$

$$\int_a^b \{m_t(\overline{m} - m) + N_t(\overline{N} - N) + w(\overline{m}_{xx} - m_{xx}) + w(\overline{N} - N)\}\, dx \geqslant 0 \tag{22}$$

$$\forall (\overline{m}, \overline{N}) \in K_2 \cap V_2;$$

$$w = w_0,\ m = m_0,\ N = N_0 \ \text{for}\ \ t = 0; \tag{23}$$

$$(m(t),\ N(t)) \in K_2 \ \text{almost everywhere} \tag{24}$$

Problem (21)-(24) is approximated by the following in the proof of this result ($\delta > 0$ is a parameter which later tends to zero)

$$w_t^\delta - m_{xx}^\delta - N^\delta = f,\quad m_t^\delta + w_{xx}^\delta + \delta^{-1}B(m^\delta, N^\delta)_1 = 0,$$

$$N_t^\delta + w^\delta + \delta^{-1}B(m^\delta, N^\delta)_2 = 0,$$

$$w^\delta = w_0,\ m^\delta = m_0,\ N^\delta = N_0 \ \text{for} \quad t = 0,$$

$$w^\delta = w_x^\delta = 0 \ \text{for} \quad x = a; \qquad m^\delta = m_x^\delta = 0 \ \text{for} \quad x = b.$$

Here $B(m^\delta, N^\delta)_i$ denotes components of the penalty operator associated with the set K_2 and acting from $[L^2(a, b)]^2$ into $[L^2(a, b)]^2$.

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